

substitution method

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coaching
calculator

data entry

$$a_1 x + b_1 y = c_1$$

$$a_2 x + b_2 y = c_2$$

x and y are variables or unknowns

a_1 and a_2 are x - coefficients

b_1 and b_2 are y - coefficients

c_1 and c_2 are constant terms

Enter the values of a_1 , b_1 and c_1 and a_2 , b_2 and c_2 on the number pad and tap the puzzled face to see the steps.

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x in terms of y or y in terms of x

x in terms of y to express x in terms of y from the chosen equation.

y in terms of x to express y in terms of x from the chosen equation.

from the first to choose the first equation

from the second to choose the second equation.

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terminology

terminology

coefficients

$$2x - 3y = 7$$

The coefficient of x is 2 and the coefficient of y is -3 .

Take the subtraction operator into account when finding the y coefficient here.

Clearly seen when we write the equation as

$$2x + (-3)y = 7$$

to read off the coefficients.

terminology

absolute value

$$2x - 3y = 4$$

$$4x + 3y = 26$$

The coefficient of y is -3 in the first equation and 3 in the second but 3 is the absolute value of both coefficients.

Think of the absolute value of a number as its distance from zero on a number line.

y -coefficients are opposite here having the same absolute value, but opposite sign.

terminology

Greatest Common Divisor

$$8x - 4y = -6$$

The coefficient of x is 8, the coefficient of y is -4 and the constant term is -6 with a GCD or Greatest Common Divisor of 2.

We often learn the GCD for positive integers, but we can extend the concept to include negative integers.

Use absolute values to find Greatest Common Divisors.

terminology

technique

technique

step-by-step

Consider for example the choices of x in terms of y and from the first.

Rearrange the first equation to express x in terms of y .

Substitute x in the second equation and solve for y .

Substitute the y solution in the rearranged first equation and solve for x .

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simplification

Occasionally one or both equations can be simplified.

$$4x - 6y = 8$$

$$2(2x - 3y) = 2(4)$$

$$2x - 3y = 4$$

The GCD of 4, -6 and 8 is 2 and we factor it out and divide both sides by it to simplify the equation.

Check for a simpler way of solving before using this technique.

technique

easy variable choice

Look for a variable that has a coefficient of $+1$ or -1 such as y in the first equation here.

$$2x - y = 9$$

$$3x + 4y = 19$$

We handle integers when solving the first equation to express y in terms of x .

It's simpler algebra when we substitute this y into the second equation.

technique

easy variable choice

Look for a variable where the absolute value of the coefficient is the same in both equations such as y here.

$$2x - 9y = 1$$

$$5x + 9y = 34$$

We stray from integer work when expressing y in terms of x from either equation, but return after substituting y into the other equation.

It's actually easier to isolate and substitute $9y$ rather than y here.

technique

harder variable choice

Look for a variable that has a coefficient that is a divisor of the coefficient of the same variable in the other equation such as y in the first equation here.

$$2x - 3y = 7$$

$$3x + 6y = 21$$

We stray from integer work when expressing y in terms of x from the first equation, but return after substituting y into the second equation.

technique

harder variable choice

What if there is no obvious choice of variable or equation?

There are no fixed rules, but in the first equation here the x coefficient is a divisor of the y coefficient.

$$2x - 6y = 7$$

$$3x + 7y = 19$$

Expressing x in terms of y from the first equation gives

$$x = \frac{7}{2} + 3y \text{ which is OK.}$$

We can always try the addition or the addition/subtraction method.

technique

back substitution

$$x - 3y = 2$$

$$x + 2y = 7$$

We can solve the first equation for

$$x = 2 + 3y$$

and then substitute this x into the second equation to get

$$(2 + 3y) + 2y = 7$$

Solve this equation to get $y = 1$ and substitute this y solution back into the rearranged first equation for

$$x = 2 + 3(1) = 5$$

technique