

addition/ subtraction method

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coaching
calculator

data entry

$$a_1 x + b_1 y = c_1$$

$$a_2 x + b_2 y = c_2$$

x and y are variables or unknowns

a_1 and a_2 are x - coefficients

b_1 and b_2 are y - coefficients

c_1 and c_2 are constant terms

Enter the values of a_1 , b_1 and c_1 and a_2 , b_2 and c_2 on the number pad and tap the puzzled face to see the steps.

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elimination

eliminate x to find y to give the x - coefficients the same absolute value.

If the x - coefficients have opposite sign add the equations to find y or subtract if the same sign.

eliminate y to find x to give the y - coefficients the same absolute value.

If the y - coefficients have opposite sign add the equations to find x or subtract if the same sign.

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LCM or smart ×

LCM to use the Least Common Multiple of the chosen variable coefficients when giving them the same absolute value.

smart × to use the chosen variable coefficients directly when giving them the same absolute value in harder cases of simultaneous equations.

first phase

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substitution

substitute in first to enter the value of the calculated variable into the first equation to find the value of the other variable.

substitute in second to enter the value of the calculated variable into the second equation to find the value of the other variable.

second phase

coaching calculator

the method

the method

the method

The ADDITION/SUBTRACTION method depends on the elimination of x or y by addition or subtraction of equations.

$$3x + y = 7$$

$$2x - 3y = 1$$

The chosen coefficients must have the **SAME ABSOLUTE VALUE** before addition or subtraction.

We could multiply the first equation by 2 and the second by 3 to give the x -coefficients the same absolute value of 6.

Multipliers are always positive.

the method

the method

The ADDITION/SUBTRACTION method depends on the elimination of x or y by addition or subtraction of equations.

$$\begin{array}{rcl} & 2x & + y = 5 \\ + & 5x & - y = 9 \\ \hline & 7x & = 14 \\ & x & = 2 \end{array}$$

The y -coefficients of 1 and -1 have the **SAME ABSOLUTE VALUE** of 1 and **OPPOSITE SIGN**.

ADD to eliminate.

the method

the method

The ADDITION/SUBTRACTION method depends on the elimination of x or y by addition or subtraction of equations.

$$\begin{array}{r} 2x + 3y = 7 \\ - \quad 2x - y = 3 \\ \hline 4y = 4 \\ y = 1 \end{array}$$

The x - coefficients of 2 have the **SAME ABSOLUTE VALUE** of 2 and **SAME SIGN**.

SUBTRACT to eliminate.

the method

terminology

terminology

coefficients

$$2x - 3y = 7$$

The coefficient of x is 2 and the coefficient of y is -3 .

Take the subtraction operator into account when finding the y coefficient here.

Clearly seen when we write the equation as

$$2x + (-3)y = 7$$

to read off the coefficients.

terminology

absolute value

$$2x - 3y = 4$$

$$4x + 3y = 26$$

The coefficient of y is -3 in the first equation and 3 in the second but 3 is the absolute value of both coefficients.

Think of the absolute value of a number as its distance from zero on a number line.

y -coefficients are opposite here having the same absolute value, but opposite sign.

terminology

opposites

$$2x - 3y = 4$$

$$4x + 3y = 26$$

The coefficient of y is -3 in the first equation and 3 in the second.

-3 is the opposite or additive inverse of 3 and 3 is the opposite of -3 .

A number and its opposite add to zero.

$$3 + (-3) = 0 \quad \text{and} \quad -3 + 3 = 0$$

Multiply a number by (-1) for its opposite.

terminology

Least Common Multiple

$$5x - 4y = 1$$

$$4x + 3y = 6$$

The coefficient of y is -4 in the first equation and 3 in the second and their LCM or their Least Common Multiple is 12 .

We often learn the LCM for positive integers, but we can extend the concept to include negative integers.

Use absolute values to find Least Common Multiples.

terminology

Greatest Common Divisor

$$8x - 4y = -6$$

The coefficient of x is 8, the coefficient of y is -4 and the constant term is -6 with a GCD or Greatest Common Divisor of 2.

We often learn the GCD for positive integers, but we can extend the concept to include negative integers.

Use absolute values to find Greatest Common Divisors.

terminology

technique

technique

simplification

Occasionally one or both equations can be simplified.

$$4x - 6y = 8$$

$$2(2x - 3y) = 2(4)$$

$$2x - 3y = 4$$

The GCD of 4, -6 and 8 is 2 and we factor it out and divide both sides by it to simplify the equation.

Check for a simpler way of solving before using this technique.

technique

easy absolute value

Look for a variable such as y here that has opposite coefficients, the SAME ABSOLUTE VALUE and OPPOSITE SIGN.

$$\begin{array}{r} 2x - 3y = 4 \\ + \quad 4x + 3y = 20 \\ \hline 6x \qquad \qquad = 24 \end{array}$$

ADD the equations.

y has gone and we can solve for x .

technique

easy absolute value

Look for a variable such as x here that has the same coefficients, the SAME ABSOLUTE VALUE and the SAME SIGN.

$$\begin{array}{r} x + 3y = 9 \\ - \quad x - 5y = 1 \\ \hline 8y = 8 \end{array}$$

SUBTRACT the equations.

$$\begin{aligned} (x + 3y) - (x - 5y) &= 9 - 1 \\ x + 3y - x + 5y &= 8 \\ 8y &= 8 \end{aligned}$$

technique

easy absolute value

Look for a variable such as y here where one coefficient is a divisor of the other.

$$\begin{array}{rcl} 5x - 4y = 6 & & 5x - 4y = 6 \\ 3x + 2y = 4 & & 3x - 2y = 4 \end{array}$$

Multiply the second equation by 2 in both cases to give the y -coefficients the SAME ABSOLUTE VALUE.

$$\begin{array}{rcl} 5x - 4y = 6 & & 5x - 4y = 6 \\ 6x + 4y = 8 & & 6x - 4y = 8 \end{array}$$

ADD in the first as OPPOSITE SIGN. SUBTRACT in the second as SAME SIGN.

technique

harder absolute value

What do we do if there are no easy absolute values?

Pick x or y and manipulate the equations to give the coefficients of our chosen variable the same absolute value.

Try to make sure that the LCM of their coefficients calculates easily when using the **LCM method** or pick easy coefficients when using the **smart $\times$ method**.

technique

harder absolute value

Using the LCM method we calculate the LCM of the coefficients of the chosen variable.

$$\begin{array}{lcl} 15x + 4y = 8 & \rightarrow & 45x + 12y = 24 \\ 12x - 6y = 5 & & 24x - 12y = 10 \end{array}$$

The LCM of the y -coefficients is 12 and we multiply the first equation by 3 and the second by 2 to give the y -coefficients the SAME ABSOLUTE VALUE, their LCM.

ADD the equations as the y -coefficients have OPPOSITE SIGN.

technique

harder absolute value

Using the smart $\times$ method we use the coefficients directly.

$$6x - 12y = 5$$

$$4x + 15y = 8$$

Here we multiply the first equation by 4 and the second by 6 to give the x -coefficients the same absolute value of 4×6 .

Much easier than multiplying the first equation by 15 and the second by 12 to give the y -coefficients the same absolute value of 15×12 .

technique

harder absolute value

Using the smart $\times$ method we normally use the coefficients directly.

$$7x - 12y = 2$$

$$4x + 3y = 1$$

Here, where the 3 is a divisor of 12, we simply multiply the second by 4 to give the y -coefficients the same, absolute value of 12.

There's no point multiplying the first by 3 and the second by 12 in easy cases such as these.

technique

back substitution

$$2x - 3y = 4$$

$$4x + 3y = 26$$

Add for $6x = 30$ and solve for $x = 5$.

Substitute this x solution back into either equation and solve for y .

With integer solutions pick an equation where the substituted variable has a small (absolute value) coefficient.

$$2(5) - 3y = 4$$

or

$$4(5) + 3y = 26$$

technique