

addition method

by Walter Kissach

coaching
calculator

data entry

$$a_1 x + b_1 y = c_1$$

$$a_2 x + b_2 y = c_2$$

x and y are variables or unknowns

a_1 and a_2 are x - coefficients

b_1 and b_2 are y - coefficients

c_1 and c_2 are constant terms

Enter the values of a_1 , b_1 and c_1 and a_2 , b_2 and c_2 on the number pad and tap the puzzled face to see the steps.

coaching calculator

elimination

eliminate x to find y to
make the x - coefficients
opposite (same absolute
value and opposite sign).

Add the equations to find y .

eliminate y to find x to
make the y - coefficients
opposite (same absolute
value and opposite sign).

Add the equations to find x .

first phase

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LCM or smart ×

LCM to use the Least Common Multiple of the chosen variable coefficients when making opposite.

smart × to use the chosen variable coefficients directly when making opposite in harder cases of simultaneous equations.

first phase

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substitution

substitute in first to enter the value of the calculated variable into the first equation to find the value of the other variable.

substitute in second to enter the value of the calculated variable into the second equation to find the value of the other variable.

second phase

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the method

the method

the method

The ADDITION method depends on the elimination of x or y by the addition of equations.

$$3x + y = 7$$

$$2x - 3y = 1$$

Before addition the chosen coefficients must be opposite having the **SAME ABSOLUTE VALUE**, but **OPPOSITE SIGN**.

We could multiply the first equation by 2 and the second by -3 to make the x -coefficients 6 and -6 respectively.

They have the same absolute value of 6, but opposite sign.

the method

the method

The ADDITION method depends on the elimination of x or y by the addition of equations.

$$\begin{array}{r} 6x + 2y = 14 \\ + \quad -6x + 9y = -3 \\ \hline 11y = 11 \\ y = 1 \end{array}$$

x - coefficients, 6 and -6 are additive inverses and sum to 0.

They are opposite having the **SAME ABSOLUTE VALUE** 6, but **OPPOSITE SIGN**.

Always **ADD**.

the method

terminology

terminology

coefficients

$$2x - 3y = 7$$

The coefficient of x is 2 and the coefficient of y is -3 .

Take the subtraction operator into account when finding the y coefficient here.

Clearly seen when we write the equation as

$$2x + (-3)y = 7$$

to read off the coefficients.

terminology

absolute value

$$2x - 3y = 4$$

$$4x + 3y = 26$$

The coefficient of y is -3 in the first equation and 3 in the second but 3 is the absolute value of both coefficients.

Think of the absolute value of a number as its distance from zero on a number line.

y -coefficients are opposite here having the same absolute value, but opposite sign.

terminology

opposite

$$2x - 3y = 4$$

$$4x + 3y = 26$$

The coefficient of y is -3 in the first equation and 3 in the second.

-3 is the opposite or additive inverse of 3 and 3 is the opposite of -3 .

A number and its opposite add to zero.

$$3 + (-3) = 0 \quad \text{and} \quad -3 + 3 = 0$$

Multiply a number by (-1) for its opposite.

terminology

Least Common Multiple

$$5x - 4y = 1$$

$$4x + 3y = 6$$

The coefficient of y is -4 in the first equation and 3 in the second and their LCM or their Least Common Multiple is 12 .

We often learn the LCM for positive integers, but we can extend the concept to include negative integers.

Use absolute values to find Least Common Multiples.

terminology

Greatest Common Divisor

$$8x - 4y = -6$$

The coefficient of x is 8, the coefficient of y is -4 and the constant term is -6 with a GCD or Greatest Common Divisor of 2.

We often learn the GCD for positive integers, but we can extend the concept to include negative integers.

Use absolute values to find Greatest Common Divisors.

terminology

technique

technique

simplification

Occasionally one or both equations can be simplified.

$$4x - 6y = 8$$

$$2(2x - 3y) = 2(4)$$

$$2x - 3y = 4$$

The GCD of 4, -6 and 8 is 2 and we factor it out and divide both sides by it to simplify the equation.

Check for a simpler way of solving before using this technique.

technique

easy opposites

The y - coefficients below are opposite, having the same absolute value, but opposite sign.

$$2x - 3y = 4$$

$$4x + 3y = 26$$

Add the equations to eliminate y .

The x - coefficients below are opposite.

$$-x + 3y = 2$$

$$x + 5y = 14$$

Add the equations to eliminate x .

technique

easy opposites

Look for a variable such as x here with the same coefficients.

$$2x + 3y = 9$$

$$2x - 5y = 1$$

Multiply either equation by -1 to change its x -coefficient to -2 .

Multiply the second equation for example to obtain opposite x -coefficients.

$$2x + 3y = 9$$

$$-2x + 5y = -1$$

Can now add the equations to eliminate x .

technique

easy opposites

Look for a variable such as y here where one coefficient is a divisor of the other.

$$\begin{array}{rcl} 5x - 4y = 6 & & 5x - 4y = 6 \\ 3x + 2y = 4 & & 3x - 2y = 4 \end{array}$$

Multiply the second equation by 2 in the first pair and by -2 in the second pair to get opposite y -coefficients.

$$\begin{array}{rcl} 5x - 4y = 6 & & 5x - 4y = 6 \\ 6x + 4y = 8 & & -6x + 4y = -8 \end{array}$$

Can now add the equations to eliminate y

technique

harder opposites

What do we do if there are no easy opposites?

Pick x or y and manipulate the equations so that the coefficients of our chosen variable are opposite (same absolute value and opposite sign).

Try to make sure that the LCM of their coefficients calculates easily when using the **LCM method** or pick easy coefficients when using the **smart $\times$ method**.

technique

harder opposites

Using the LCM method we calculate the LCM of the coefficients of the chosen variable.

$$6x - 12y = 5$$

$$4x + 15y = 8$$

The LCM of the x - coefficients is 12 and we multiply the first equation by 2 and the second by -3 or the first by -2 and the second by 3 for opposite x - coefficients.

The LCM of 6 and 4 is much easier than -12 and 15.

technique

harder opposites

Using the smart $\times$ method we use the coefficients directly.

$$6x - 12y = 5$$

$$4x + 15y = 8$$

Here we multiply the first equation by 4 and the second equation by -6 to make the x -coefficients opposite, 24 and -24 respectively.

Much easier than multiplying the first equation by 15 and the second by 12 to make the y -coefficients opposite.

technique

harder opposites

Using the smart $\times$ method we normally use the coefficients directly.

$$7x - 12y = 2$$

$$4x + 3y = 1$$

Here, where the 3 is a divisor of 12, we simply multiply the second by 4 to make the y -coefficients opposite, -12 and 12 respectively.

There's no point multiplying the first by 3 and the second by 12 in easy cases such as these.

technique

back substitution

$$2x - 3y = 4$$

$$4x + 3y = 26$$

Add for $6x = 30$ and solve for $x = 5$.

Substitute this x solution back into either equation and solve for y .

With integer solutions pick an equation where the substituted variable has a small (absolute value) coefficient.

$$2(5) - 3y = 4$$

or

$$4(5) + 3y = 26$$

technique