

quadratic
trinomials

inspection

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terminology

terminology

quadratic polynomial
monomial

$$x^2$$

$$6x^2$$

binomial

$$3x^2 - 12$$

$$5x^2 + 2x$$

trinomial

$$x^2 + 4x - 5$$

$$x^2 - 5x + 6$$

$$5x^2 - x - 6$$

terminology

terms and coefficients

Standard form $ax^2 + bx + c$.

The quadratic trinomial $2x^2 - 7x + 5$ has three terms $2x^2$, $-7x$ and 5 .

2 , -7 and 5 , the a , b and c , are the coefficients of the x^2 , x and constant terms (constant term is also 5).

The x term is $-7x$ with a coefficient of (-7) .

Take account of the $+$ or the $-$ operator when finding coefficients.

terminology

discriminant

standard form :

$$ax^2 + bx + c, a \neq 0$$

discriminant :

$$D = b^2 - 4ac$$

factors when :

$$D \geq 0$$

The expression factors over the integers with linear factors such as $(2x - 3)$ when D is a perfect square.

The focus is on expressions where a , b and c are integers here.

terminology

technique

technique

descending powers of x

We often prefer to work with polynomials in descending powers of the variable.

If we receive a polynomial in ascending powers we can reorder it in descending powers of the variable.

$$12 - 10x - 2x^2 = -2x^2 - 10x + 12$$

$$1 - 5x - x^2 = -x^2 - 5x + 6$$

We can, of course, leave the polynomial in ascending powers when appropriate.

technique

common factor

One of the first steps in factorizing any polynomial is to factor out a common numerical factor.

Factor out the greatest common numerical factor where it makes sense.

When the first term has a negative coefficient we also factor out negative one .

If there is a variable common to all terms, factor out one with the lowest exponent.

technique

when to use

Factorize by inspection when a quadratic trinomial has a coefficient of x^2 of one.

$$x^2 + 5x - 6$$

$$2x^2 + 10x - 12 = 2(x^2 + 5x - 6)$$

$$\begin{aligned} 18 - 15x - 3x^2 &= -3x^2 - 15x + 18 \\ &= (-3)(x^2 + 5x - 6) \end{aligned}$$

$$\begin{aligned} 6 - 5x - x^2 &= -x^2 - 5x + 6 \\ &= (-1)(x^2 + 5x - 6) \end{aligned}$$

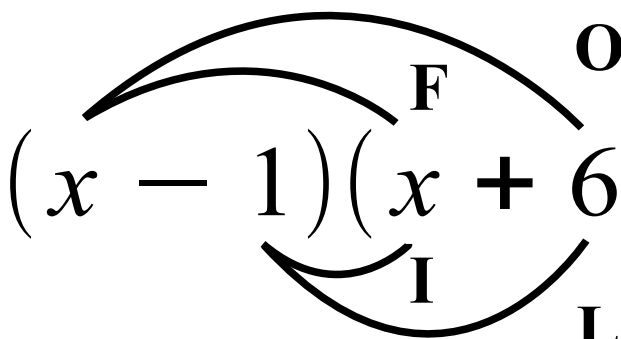
technique

factorization check

distribute to check

$$\begin{aligned}(x - 1)(x + 6) &= x(x + 6) - 1(x + 6) \\ &= x^2 + 6x - x - 6 \\ &= x^2 + 5x - 6\end{aligned}$$

or check with **FOIL**


$$\begin{aligned}(x - 1)(x + 6) &= x^2 + 6x - x - 6 \\ &= x^2 + 5x - 6\end{aligned}$$

F irst $(x)(x) = x^2$

O uter $(x)(6) = 6x$

I nner $(-1)(x) = -x$

L ast $(-1)(6) = -6$

technique

examples

Focus is on quadratic trinomials following appropriate factoring out of the greatest numerical factor.

positive constant term

product and sum

$$x^2 + 13x + 12$$

coefficient of x is (13)

constant term is (12)

we need two integers with :

product of (12), sum (13)

+ of (+12) means same signs

+ of (+13) means both positive

$$(1)(12) = 12$$

$$(1) + (12) = 13$$

examples

...

tabulate (optional)

two integers with :
product (12), sum (13)

same signs
both positive

1	12	stop the tabulation when sum achieved
2	6	
3	4	

$$(1)(12) = 12$$
$$(1) + (12) = 13$$

factorize

$$x^2 + 13x + 12 = (x + 1)(x + 12)$$

examples

positive constant term

product and sum

$$x^2 - 11x + 18$$

coefficient of x is (-11)

constant term is (18)

we need two integers with :
product of (18) , sum (-11)

$+$ of $(+18)$ means same signs

$-$ of (-11) means both negative

$$(-2)(-9) = 18$$

$$(-2) + (-9) = -11$$

examples

...

tabulate (optional)

two integers with :

product (18), sum (− 11)

same signs

both negative

− 1 − 18

− 2 − 9 stop the tabulation
when sum achieved

− 3 − 6

$$(-2)(-9) = 18$$

$$(-2) + (-9) = -11$$

factorize

$$x^2 - 11x + 18 = (x - 2)(x - 9)$$

examples

negative constant term

product and sum

$$x^2 + 5x - 36$$

coefficient of x is (5)

constant term is (-36)

we need two integers with :

product of (-36) , sum (5)

— of (-36) means opposite signs

+ of $(+5)$ means the one further from zero is positive

$$(-4)(9) = -36$$

$$(-4) + (9) = 5$$

examples

...

tabulate (optional)

two integers with :

product (-36) , sum (5)

opposite signs

one further from zero is positive

-1	36	skip early calculations
-2	18	when you think target
-3	12	sum is a later entry
-4	9	stop the tabulation
-6	6	when sum achieved

$$(-4)(9) = -36$$

$$(-4) + (9) = 5$$

factorize

$$x^2 + 5x - 36 = (x - 4)(x + 9)$$

examples

negative constant term

product and sum

$$x^2 - x - 42$$

coefficient of x is (-1)

constant term is (-42)

we need two integers with :
product of (-42) , sum (-1)

— of (-42) means opposite signs

— of (-1) means the one further
from zero is negative

$$(6)(-7) = -42$$

$$(6) + (-7) = -1$$

examples

...

tabulate (optional)

two integers with :

product (-42) , sum (-1)

opposite signs

one further from zero is negative

1 -42

2 -21

3 -14

6 **-7**

$$(6)(-7) = -42$$

$$(6) + (-7) = -1$$

factorize

$$x^2 - x - 42 = (x + 6)(x - 7)$$

examples