

quadratic trinomials

guess and check

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terminology

terminology

quadratic polynomial
monomial

$$x^2$$

$$6x^2$$

binomial

$$3x^2 - 12$$

$$5x^2 + 2x$$

trinomial

$$x^2 + 4x - 5$$

$$2x^2 - 7x + 6$$

$$5x^2 - x - 6$$

terminology

terms and coefficients

Standard form $ax^2 + bx + c$.

The quadratic trinomial $2x^2 - 7x + 5$ has three terms $2x^2$, $-7x$ and 5 .

2 , -7 and 5 , the a , b and c , are the coefficients of the x^2 , x and constant terms (constant term is also 5).

The x term is $-7x$ with a coefficient of (-7) .

Take account of the $+$ or the $-$ operator when finding coefficients.

terminology

discriminant

standard form :

$$ax^2 + bx + c, a \neq 0$$

discriminant :

$$D = b^2 - 4ac$$

factors when :

$$D \geq 0$$

The expression factors over the integers with linear factors such as $(2x - 3)$ when D is a perfect square.

The focus is on expressions where a , b and c are integers here.

terminology

technique

technique

descending powers of x

We often prefer to work with polynomials in descending powers of the variable.

If we receive a polynomial in ascending powers we can reorder it in descending powers of the variable.

$$12 - 10x - 2x^2 = -2x^2 - 10x + 12$$

$$1 - 5x - 6x^2 = -6x^2 - 5x + 6$$

We can, of course, leave the polynomial in ascending powers when appropriate.

technique

common factor

One of the first steps in factorizing any polynomial is to factor out a common numerical factor.

Factor out the greatest common numerical factor where it makes sense.

When the first term has a negative coefficient we also factor out negative one .

If there is a variable common to all terms, factor out one with the lowest exponent.

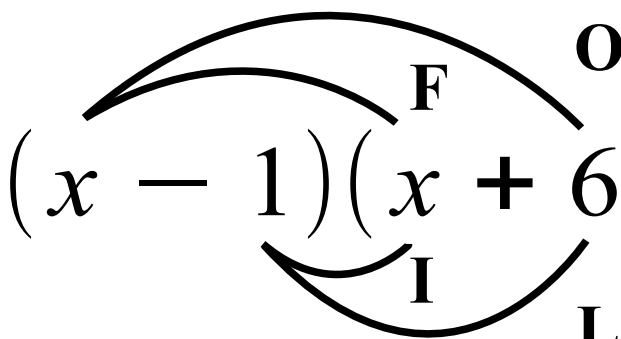
technique

factorization check

distribute to check

$$\begin{aligned}(x - 1)(x + 6) &= x(x + 6) - 1(x + 6) \\ &= x^2 + 6x - x - 6 \\ &= x^2 + 5x - 6\end{aligned}$$

or check with **FOIL**


$$\begin{aligned}(x - 1)(x + 6) &= x^2 + 6x - x - 6 \\ &= x^2 + 5x - 6\end{aligned}$$

F irst $(x)(x) = x^2$

O uter $(x)(6) = 6x$

I nner $(-1)(x) = -x$

L ast $(-1)(6) = -6$

technique

examples

Focus is on quadratic trinomials following appropriate factoring out of the greatest numerical factor.

positive constant term

products and signs

$$2x^2 + 9x + 4$$

coefficient of x^2 is (2)

coefficient of x is (9)

constant term is (4)

we need two positive integers
product of (2)

we need two integers with
product of (4)

+ of (+4) means same signs

+ of (+9) means both positive

examples

...

build tables

$$2x^2 + 9x + 4$$

two positive integers, product (2)

two integers, product (4)

both positive

1	2	1	4	4	1
		2	2		

check each red pair with each blue pair where sensible

examples

...

check

$$2x^2 + 9x + 4$$

1 2 with 1 4 or 4 1

$$(x + 1)(2x + 4)$$

⊘ second factor factorizes

$$(x + 4)(2x + 1)$$

😊 $x(1) + 4(2x) = 9x$

sum the terms in x (outers and
innners) until we get $(9x)$

$$2x^2 + 9x + 4 = (x + 4)(2x + 1)$$

examples

positive constant term

products and signs

$$6x^2 - 11x + 3$$

coefficient of x^2 is (6)

coefficient of x is (-11)

constant term is (3)

we need two positive integers
product of (6)

we need two integers with
product of (3)

+ of (+3) means same signs

— of (-11) means both negative

examples

...

build tables

$$6x^2 - 11x + 3$$

two positive integers, product (6)

two integers, product (3)

both negative

1	6	−1	−3	−3	−1
2	3				

check each red pair with each blue pair where sensible

examples

...

check

$$6x^2 - 11x + 3$$

1 6 with -1 -3 or -3 -1

$$(x - 1)(6x - 3)$$

⊘ second factor factorizes

$$(x - 3)(6x - 1)$$

$$\text{⊘ } x(-1) - 3(6x) = -19x$$

sum the terms in x (outers and
innors) until we get $(-11x)$

examples

...

check

$$6x^2 - 11x + 3$$

2 3 with -1 -3 or -3 -1

$$(2x - 1)(3x - 3)$$

⊘ second factor factorizes

$$(2x - 3)(3x - 1)$$

😊 $2x(-1) - 3(3x) = -11x$

sum the terms in x (outers and
innners) until we get $(-11x)$

$$6x^2 - 11x + 3 = (2x - 3)(3x - 1)$$

examples

negative constant term

products and signs

$$6x^2 + 5x - 6$$

coefficient of x^2 is (6)

coefficient of x is (5)

constant term is (-6)

we need two positive integers
product of (6)

we need two integers with
product of (-6)

— of (-6) means opposite signs

examples

...

build tables

$$6x^2 + 5x - 6$$

two positive integers, product (6)

two integers, product (−6)
opposite signs

1	6	1	−6	−6	1
2	3	2	−3	−3	2

don't tabulate pairs with signs swapped such as −1 and 6 as we check automatically

check each red pair with each blue pair where sensible

examples

...

check

$$6x^2 + 5x - 6$$

$$1 \quad 6 \text{ with } 1 \quad -6 \text{ or } -6 \quad 1$$

$$(x + 1)(6x - 6)$$

⊘ second factor factorizes

$$(x - 6)(6x + 1)$$

$$\text{⊘ } x(1) - 6(6x) = -35x$$

don't check **pairs** with signs swapped as we reject similarly

sum the terms in x (outers and inners) until we get $(5x)$

examples

...

check

$$6x^2 + 5x - 6$$

1 6 with 2 -3 or -3 2

$$(x + 2)(6x - 3)$$

⊘ second factor factorizes

$$(x - 3)(6x + 2)$$

⊘ second factor factorizes

don't check **pairs** with signs swapped as we reject similarly

sum the terms in x (outers and inners) until we get $(5x)$

examples

...

check

$$6x^2 + 5x - 6$$

$$2 \quad 3 \text{ with } 1 \quad -6 \text{ or } -6 \quad 1$$

$$(2x + 1)(3x - 6)$$

⊘ second factor factorizes

$$(2x - 6)(3x + 1)$$

⊘ first factor factorizes

don't check **pairs** with signs swapped as we reject similarly

sum the terms in x (outers and inners) until we get $(5x)$

examples

...

check

$$6x^2 + 5x - 6$$

$$2 \quad 3 \text{ with } 2 \quad -3 \text{ or } -3 \quad 2$$

$$(2x + 2)(3x - 3)$$

 both factors factorize

$$(2x - 3)(3x + 2)$$

 $2x(2) - 3(3x) = -5x$

it may not be $(5x)$, but we can swap signs

$$(2x + 3)(3x - 2)$$

 $2x(-2) + 3(3x) = 5x$

$$6x^2 + 5x - 6 = (2x + 3)(3x - 2)$$

examples

negative constant term

products and signs

$$5x^2 - 3x - 14$$

coefficient of x^2 is (5)

coefficient of x is (-3)

constant term is (-14)

we need two positive integers
product of (5)

we need two integers with
product of (-14)

— of (-14) means opposite signs

examples

...

build tables

$$5x^2 - 3x - 14$$

two positive integers, product (5)

two integers, product (−14)
opposite signs

1	5	1	−14	−14	1
		2	−7	−7	2

don't tabulate pairs with signs swapped such as −1 and 14 as we check automatically

check each red pair with each blue pair where sensible

examples

...

check

$$5x^2 - 3x - 14$$

$$1 \quad 5 \text{ with } 1 \quad -14 \text{ or } -14 \quad 1$$

$$(x + 1)(5x - 14)$$

$$\textcircled{\times} x(-14) + 1(5x) = -9x$$

$$(x - 14)(5x + 1)$$

$$\textcircled{\times} x(1) - 14(5x) = -69x$$

don't check **pairs** with signs swapped as we reject similarly

sum the terms in x (outers and inners) until we get $(-3x)$

examples

...

check

$$5x^2 - 3x - 14$$

1 5 with 2 -7 or -7 -2

$$(x + 2)(5x - 7)$$

 $x(-7) + 2(5x) = 3x$

it may not be $(-3x)$, but we can swap signs

$$(x - 2)(5x + 7)$$

 $x(7) - 2(5x) = -3x$

sum the terms in x (outers and inners) until we get $(-3x)$

$$5x^2 - 3x - 14 = (x - 2)(5x + 7)$$

examples