

quadratic trinomials

grouping

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terminology

terminology

quadratic polynomial
monomial

$$x^2$$

$$6x^2$$

binomial

$$3x^2 - 12$$

$$5x^2 + 2x$$

trinomial

$$x^2 + 4x - 5$$

$$2x^2 - 7x + 6$$

$$5x^2 - x - 6$$

terminology

terms and coefficients

Standard form $ax^2 + bx + c$.

The quadratic trinomial $2x^2 - 7x + 5$ has three terms $2x^2$, $-7x$ and 5 .

2 , -7 and 5 , the a , b and c , are the coefficients of the x^2 , x and constant terms (constant term is also 5).

The x term is $-7x$ with a coefficient of (-7) .

Take account of the $+$ or the $-$ operator when finding coefficients.

terminology

discriminant

standard form :

$$ax^2 + bx + c, a \neq 0$$

discriminant :

$$D = b^2 - 4ac$$

factors when :

$$D \geq 0$$

The expression factors over the integers with linear factors such as $(2x - 3)$ when D is a perfect square.

The focus is on expressions where a , b and c are integers here.

terminology

technique

technique

descending powers of x

We often prefer to work with polynomials in descending powers of the variable.

If we receive a polynomial in ascending powers we can reorder it in descending powers of the variable.

$$12 - 10x - 2x^2 = -2x^2 - 10x + 12$$

$$1 - 5x - 6x^2 = -6x^2 - 5x + 6$$

We can, of course, leave the polynomial in ascending powers when appropriate.

technique

common factor

One of the first steps in factorizing any polynomial is to factor out a common numerical factor.

Factor out the greatest common numerical factor where it makes sense.

When the first term has a negative coefficient we also factor out negative one .

If there is a variable common to all terms, factor out one with the lowest exponent.

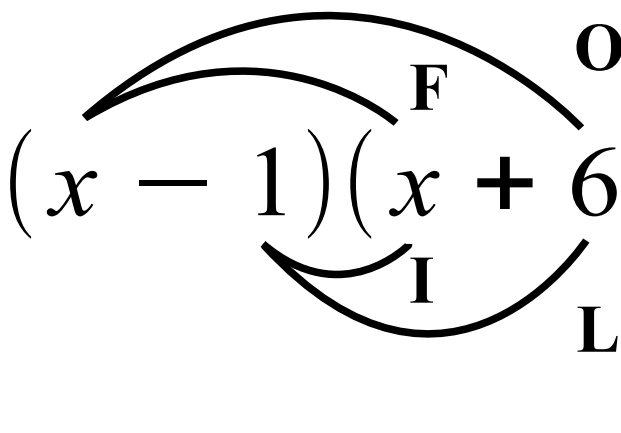
technique

factorization check

distribute to check

$$\begin{aligned}(x - 1)(x + 6) &= x(x + 6) - 1(x + 6) \\ &= x^2 + 6x - x - 6 \\ &= x^2 + 5x - 6\end{aligned}$$

or check with **FOIL**


$$\begin{aligned}(x - 1)(x + 6) &= x^2 + 6x - x - 6 \\ &= x^2 + 5x - 6\end{aligned}$$

F irst $(x)(x) = x^2$

O uter $(x)(6) = 6x$

I nner $(-1)(x) = -x$

L ast $(-1)(6) = -6$

technique

examples

Focus is on quadratic trinomials following appropriate factoring out of the greatest numerical factor.

positive constant term

product and sum

$$2x^2 + 9x + 4$$

coefficient of x^2 is (2)

coefficient of x is (9)

constant term is (4)

we need two integers with :

product of (2)(4) = 8

sum of (9)

+ of (+4) means same signs

+ of (+9) means both positive

$$(1)(8) = 8$$

$$(1) + (8) = 9$$

examples

...

tabulate (optional)

two integers with :

product (8), sum (9)

same signs

both positive

1 8 stop the tabulation
2 4 when sum achieved

$$(1)(8) = 8$$

$$(1) + (8) = 9$$

examples

...

rewrite the x term

$$2x^2 + 9x + 4$$

$$= 2x^2 + x + 8x + 4$$

can also have

$$2x^2 + 9x + 4$$

$$= 2x^2 + 8x + x + 4$$

recall our choice is (1) and (8)

examples

...

group and factor

$$2x^2 + 9x + 4$$

$$= 2x^2 + x + 8x + 4$$

$$= (2x^2 + x) + (8x + 4)$$

$$= x(2x + 1) + 4(2x + 1)$$

examples

...

factor out common factor

$$2x^2 + 9x + 4$$

$$= 2x^2 + x + 8x + 4$$

$$= (2x^2 + x) + (8x + 4)$$

$$= x(2x + 1) + 4(2x + 1)$$

$$= (2x + 1)(x + 4)$$

examples

positive constant term

product and sum

$$3x^2 - 14x + 15$$

coefficient of x^2 is (3)

coefficient of x is (-14)

constant term is (15)

we need two integers with :

product of $(3)(15) = 45$

sum of (-14)

$+$ of $(+15)$ means same signs

$-$ of (-14) means both negative

$$(-5)(-9) = 45$$

$$(-5) + (-9) = -14$$

examples

...

tabulate (optional)

two integers with :

product (45), sum (−14)

same signs

both negative

−1 −45

−3 −15

−5 −9

$$(-5)(-9) = 45$$

$$(-5) + (-9) = -14$$

examples

...

rewrite the x term

$$\begin{aligned} 3x^2 - 14x + 15 \\ = 3x^2 - 5x - 9x + 15 \end{aligned}$$

can also have

$$\begin{aligned} 3x^2 - 14x + 15 \\ = 3x^2 - 9x - 5x + 15 \end{aligned}$$

recall our choice is (-5) and (-9)

examples

...

group and factor

$$3x^2 - 14x + 15$$

$$= 3x^2 - 5x - 9x + 15$$

$$= (3x^2 - 5x) - (9x - 15)$$

$$= x(3x - 5) - 3(3x - 5)$$

examples

...

factor out common factor

$$\begin{aligned} & 3x^2 - 14x + 15 \\ &= 3x^2 - 5x - 9x + 15 \\ &= (3x^2 - 5x) - (9x - 15) \\ &= x(3x - 5) - 3(3x - 5) \\ &= (3x - 5)(x - 3) \end{aligned}$$

examples

negative constant term

product and sum

$$6x^2 + 5x - 6$$

coefficient of x^2 is (6)

coefficient of x is (5)

constant term is (-6)

we need two integers with :

product of $(6)(-6) = -36$

sum of (5)

— of (-6) means opposite signs

+ of $(+5)$ means the one further from zero is positive

$$(-4)(9) = -36$$

$$(-4) + (9) = 5$$

examples

...

tabulate (optional)

two integers with :

product (-36) , sum (5)

opposite signs

one further from zero is positive

-1	36	skip early calculations
-2	18	when you think target
-3	12	sum is a later entry
-4	9	stop the tabulation
-6	6	when sum achieved

$$(-4)(9) = -36$$

$$(-4) + (9) = 5$$

examples

...

rewrite the x term

$$6x^2 + 5x - 6$$

$$= 6x^2 - 4x + 9x - 6$$

can also have

$$6x^2 + 5x - 6$$

$$= 6x^2 + 9x - 4x - 6$$

recall our choice is (-4) and (9)

examples

...

group and factor

$$6x^2 + 5x - 6$$

$$= 6x^2 - 4x + 9x - 6$$

$$= (6x^2 - 4x) + (9x - 6)$$

$$= 2x(3x - 2) + 3(3x - 2)$$

examples

...

factor out common factor

$$\begin{aligned} &6x^2 + 5x - 6 \\ &= 6x^2 - 4x + 9x - 6 \\ &= (6x^2 - 4x) + (9x - 6) \\ &= 2x(3x - 2) + 3(3x - 2) \\ &= (3x - 2)(2x + 3) \end{aligned}$$

examples

negative constant term

product and sum

$$6x^2 - x - 12$$

coefficient of x^2 is (6)

coefficient of x is (-1)

constant term is (-12)

we need two integers with :

product of $(6)(-12) = -72$

sum of (-1)

— of (-12) means opposite signs

— of (-1) means the one further from zero is negative

$$(8)(-9) = -72$$

$$(8) + (-9) = -1$$

examples

...

tabulate (optional)

two integers with :

product (-72) , sum (-1)

opposite signs

one further from zero is negative

1 -72

2 -36

3 -24

4 -18

6 -12

8 -9

$$(8)(-9) = -72$$

$$(8) + (-9) = -1$$

examples

...

rewrite the x term

$$6x^2 - x - 12$$

$$= 6x^2 + 8x - 9x - 12$$

can also have

$$6x^2 - x - 12$$

$$= 6x^2 - 9x + 8x - 12$$

recall our choice is (8) and (-9)

examples

...

group and factor

$$6x^2 - x - 12$$

$$= 6x^2 + 8x - 9x - 12$$

$$= (6x^2 + 8x) - (9x + 12)$$

$$= 2x(3x + 4) - 3(3x + 4)$$

examples

...

factor out common factor

$$6x^2 - x - 12$$

$$= 6x^2 + 8x - 9x - 12$$

$$= (6x^2 + 8x) - (9x + 12)$$

$$= 2x(3x + 4) - 3(3x + 4)$$

$$= (3x + 4)(2x - 3)$$

examples