

quadratic trinomials

grid or box

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terminology

terminology

quadratic polynomial
monomial

$$x^2$$

$$6x^2$$

binomial

$$3x^2 - 12$$

$$5x^2 + 2x$$

trinomial

$$x^2 + 4x - 5$$

$$2x^2 - 7x + 6$$

$$5x^2 - x - 6$$

terminology

terms and coefficients

Standard form $ax^2 + bx + c$.

The quadratic trinomial $2x^2 - 7x + 5$ has three terms $2x^2$, $-7x$ and 5 .

2 , -7 and 5 , the a , b and c , are the coefficients of the x^2 , x and constant terms (constant term is also 5).

The x term is $-7x$ with a coefficient of (-7) .

Take account of the $+$ or the $-$ operator when finding coefficients.

terminology

discriminant

standard form :

$$ax^2 + bx + c, a \neq 0$$

discriminant :

$$D = b^2 - 4ac$$

factors when :

$$D \geq 0$$

The expression factors over the integers with linear factors such as $(2x - 3)$ when D is a perfect square.

The focus is on expressions where a , b and c are integers here.

terminology

technique

technique

descending powers of x

We often prefer to work with polynomials in descending powers of the variable.

If we receive a polynomial in ascending powers we can reorder it in descending powers of the variable.

$$12 - 10x - 2x^2 = -2x^2 - 10x + 12$$

$$1 - 5x - 6x^2 = -6x^2 - 5x + 6$$

We can, of course, leave the polynomial in ascending powers when appropriate.

technique

common factor

One of the first steps in factorizing any polynomial is to factor out a common numerical factor.

Factor out the greatest common numerical factor where it makes sense.

When the first term has a negative coefficient we also factor out negative one .

If there is a variable common to all terms, factor out one with the lowest exponent.

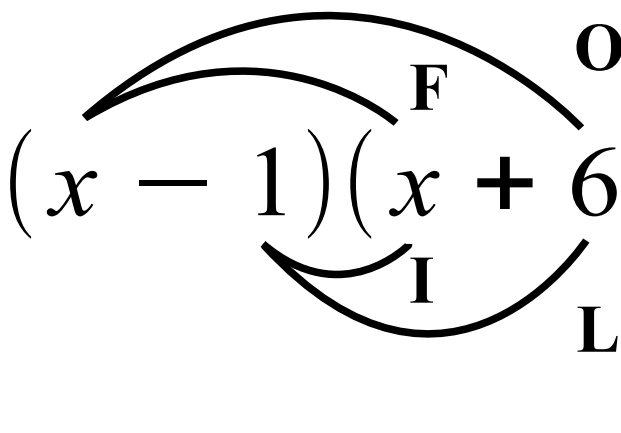
technique

factorization check

distribute to check

$$\begin{aligned}(x - 1)(x + 6) &= x(x + 6) - 1(x + 6) \\ &= x^2 + 6x - x - 6 \\ &= x^2 + 5x - 6\end{aligned}$$

or check with **FOIL**


$$\begin{aligned}(x - 1)(x + 6) &= x^2 + 6x - x - 6 \\ &= x^2 + 5x - 6\end{aligned}$$

F first $(x)(x) = x^2$

O uter $(x)(6) = 6x$

I nner $(-1)(x) = -x$

L ast $(-1)(6) = -6$

technique

examples

Focus is on quadratic trinomials following appropriate factoring out of the greatest numerical factor.

positive constant term

draw the box

$$2x^2 + 9x + 4$$

$2x^2$	
	$+4$

examples

...

product and sum

$$2x^2 + 9x + 4$$

coefficient of x^2 is (2)

coefficient of x is (9)

constant term is (4)

we need two integers with :

product of (2)(4) = 8

sum of (9)

+ of (+4) means same signs

+ of (+9) means both positive

$$(1)(8) = 8$$

$$(1) + (8) = 9$$

examples

...

tabulate (optional)

two integers with :

product (8), sum (9)

same signs

both positive

1 8 stop the tabulation
2 4 when sum achieved

$$(1)(8) = 8$$

$$(1) + (8) = 9$$

examples

...

complete the box

$$2x^2 + 9x + 4$$

$(1) + (8) = 9$ and we have
 $(+1x) + (+8x) = +9x$

$2x^2$	$+1x$
$+8x$	$+4$

can swap $(+1x)$ and $(+8x)$

examples

...

factor columns and rows

$$2x^2 + 9x + 4$$

	$2x$	$+1$
x	$2x^2$	$+1x$
$+4$	$+8x$	$+4$

match red signs
match blue signs

factor out 1 or -1 appropriately
if no other common factor

examples

...

read off the factors

$$2x^2 + 9x + 4$$
$$= (2x + 1)(x + 4)$$

	$2x$	$+1$
x	$2x^2$	$+1x$
$+4$	$+8x$	$+4$

examples

positive constant term

draw the box

$$3x^2 - 14x + 15$$

$3x^2$	
	$+15$

examples

...

product and sum

$$3x^2 - 14x + 15$$

coefficient of x^2 is (3)

coefficient of x is (-14)

constant term is (15)

we need two integers with :

product of (3)(15) = 45

sum of (-14)

+ of (+15) means same signs

- of (-14) means both negative

$$(-5)(-9) = 45$$

$$(-5) + (-9) = -14$$

examples

...

tabulate (optional)

two integers with :

product (45), sum (−14)

same signs

both negative

−1 −45

−3 −15

−5 −9

$$(-5)(-9) = 45$$

$$(-5) + (-9) = -14$$

examples

...

complete the box

$$3x^2 - 14x + 15$$

$(-5) + (-9) = -14$ and we have

$$(-5x) + (-9x) = -14x$$

$3x^2$	$-5x$
$-9x$	$+15$

can swap $(-5x)$ and $(-9x)$

examples

...

factor columns and rows

$$3x^2 - 14x + 15$$

	$3x$	-5
x	$3x^2$	$-5x$
-3	$-9x$	$+15$

match red signs
match blue signs

examples

...

read off the factors

$$3x^2 - 14x + 15$$
$$= (3x - 5)(x - 3)$$

	$3x$	-5
x	$3x^2$	$-5x$
-3	$-9x$	$+15$

examples

negative constant term

draw the box

$$6x^2 + 5x - 6$$

$6x^2$	
	-6

examples

...

product and sum

$$6x^2 + 5x - 6$$

coefficient of x^2 is (6)

coefficient of x is (5)

constant term is (−6)

we need two integers with :

product of (6)(−6) = −36

sum of (5)

− of (−6) means opposite signs

+ of (+5) means the one further from zero is positive

$$(-4)(9) = -36$$

$$(-4) + (9) = 5$$

examples

...

tabulate (optional)

two integers with :

product (-36) , sum (5)

opposite signs

one further from zero is positive

-1	36	skip early calculations
-2	18	when you think target
-3	12	sum is a later entry
-4	9	stop the tabulation
-6	6	when sum achieved

$$(-4)(9) = -36$$

$$(-4) + (9) = 5$$

examples

...

complete the box

$$6x^2 + 5x - 6$$

$(-4) + (9) = 5$ and we have

$$(-4x) + (+9x) = +5x$$

$6x^2$	$-4x$
$+9x$	-6

can swap $(-4x)$ and $(+9x)$

examples

...

factor columns and rows

$$6x^2 + 5x - 6$$

	$3x$	-2
$2x$	$6x^2$	$-4x$
$+3$	$+9x$	-6

match red signs
match blue signs

examples

...

read off the factors

$$6x^2 + 5x - 6$$
$$= (3x - 2)(2x + 3)$$

	$3x$	-2
$2x$	$6x^2$	$-4x$
$+3$	$+9x$	-6

examples

negative constant term

draw the box

$$6x^2 - x - 12$$

$6x^2$	
	-12

examples

...

product and sum

$$6x^2 - x - 12$$

coefficient of x^2 is (6)

coefficient of x is (-1)

constant term is (-12)

we need two integers with :

product of $(6)(-12) = -72$

sum of (-1)

— of (-12) means opposite signs

— of (-1) means the one further from zero is negative

$$(8)(-9) = -72$$

$$(8) + (-9) = -1$$

examples

...

tabulate (optional)

two integers with :

product (-72) , sum (-1)

opposite signs

one further from zero is negative

1 -72

2 -36

3 -24

4 -18

6 -12

8 -9

$$(8)(-9) = -72$$

$$(8) + (-9) = -1$$

examples

...

complete the box

$$6x^2 - x - 12$$

$(8) + (-9) = -1$ and we have

$$(+8x) + (-9x) = -x$$

$6x^2$	$+8x$
$-9x$	-12

can swap $(+8x)$ and $(-9x)$

examples

...

factor columns and rows

$$6x^2 - x - 12$$

	$3x$	$+4$
$2x$	$6x^2$	$+8x$
-3	$-9x$	-12

match red signs
match blue signs

examples

...

read off the factors

$$6x^2 - x - 12 = (3x + 4)(2x - 3)$$

	$3x$	$+4$
$2x$	$6x^2$	$+8x$
-3	$-9x$	-12

examples