

quadratic binomials

$$ax^2 + b$$

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terminology

terminology

quadratic polynomials

monomial

$$x^2$$

$$6x^2$$

binomial

$$3x^2 - 12$$

$$8 - 18x^2$$

$$2x^2 + 3$$

trinomial

$$x^2 + 4x - 5$$

$$5x^2 - x - 6$$

terminology

terms and coefficients

The quadratic binomial $ax^2 + b$ is in descending powers of the variable x .

The quadratic binomial $2x^2 - 8$ has two terms $2x^2$ and -8 .

2 and -8 are the coefficients of the x^2 and constant terms.

The constant term is -8 .

Take account of the $+$ or the $-$ operator when finding coefficients.

terminology

technique

technique

descending powers of x

We often prefer to work with polynomials in descending powers of the variable.

If we receive a polynomial in ascending powers we can reorder it in descending powers of the variable.

$$-9 + 4x^2 = 4x^2 - 9$$

$$1 - 4x^2 = -4x^2 + 1$$

We can, of course, leave the polynomial in ascending powers when appropriate.

technique

common factor

One of the first steps in factorizing any polynomial is to factor out a common numerical factor.

Factor out the greatest common numerical factor where it makes sense.

When the first term has a negative coefficient we also factor out negative one .

If there is a variable common to all terms, factor out one with the lowest exponent.

technique

examples

quadratic binomials

$$\begin{aligned}x^2 - 36 &= (x)^2 - (6)^2 \\ &= (x + 6)(x - 6)\end{aligned}$$

$$\begin{aligned}16 - x^2 &= (4)^2 - (x)^2 \\ &= (4 + x)(4 - x)\end{aligned}$$

write each term as squares
and factor with the difference
of squares

$$\begin{aligned}a^2 - b^2 &= (a + b)(a - b) \\ &= (a - b)(a + b)\end{aligned}$$

note that the coefficients of the
 x^2 term and constant term have
different sign

examples

quadratic binomials

$$\begin{aligned}49 - x^2 &= (7)^2 - (x)^2 \\ &= (7 + x)(7 - x)\end{aligned}$$

$$\begin{aligned}49 - x^2 &= -x^2 + 49 \\ &= -(x^2 - 49) \\ &= -((x)^2 - (7)^2) \\ &= -(x + 7)(x - 7)\end{aligned}$$

in the last case optionally reorder
in descending powers of x and
factor out (-1)

write each term as squares and
factor with the difference of
squares

examples

quadratic binomials

$$\begin{aligned}x^2 - 1 &= (x)^2 - (1)^2 \\&= (x + 1)(x - 1)\end{aligned}$$

$$\begin{aligned}4x^2 - 9 &= (2x)^2 - (3)^2 \\&= (2x + 3)(2x - 3)\end{aligned}$$

$$\begin{aligned}9x^2 - 1 &= (3x)^2 - (1)^2 \\&= (3x + 1)(3x - 1)\end{aligned}$$

$$\begin{aligned}25 - 4x^2 &= (5)^2 - (2x)^2 \\&= (5 + 2x)(5 - 2x)\end{aligned}$$

$$\begin{aligned}1 - 16x^2 &= (1)^2 - (4x)^2 \\&= (1 + 4x)(1 - 4x)\end{aligned}$$

examples

quadratic binomials

$$\begin{aligned}50x^2 - 8 &= 2(25x^2 - 4) \\&= 2((5x)^2 - (2)^2) \\&= 2(5x + 2)(5x - 2)\end{aligned}$$

$$\begin{aligned}12 - 27x^2 &= 3(4 - 9x^2) \\&= 3((2)^2 - (3x)^2) \\&= 3(2 + 3x)(2 - 3x)\end{aligned}$$

$$\begin{aligned}12 - 27x^2 &= -27x^2 + 12 \\&= (-3)(9x^2 - 4) \\&= (-3)((3x)^2 - (2)^2) \\&= (-3)(3x + 2)(3x - 2)\end{aligned}$$

examples

mental workout

easy

$$\begin{aligned} 12 \times 8 &= (10 + 2)(10 - 2) \\ &= 10^2 - 2^2 \\ &= 96 \end{aligned}$$

$$\begin{aligned} 9 \times 11 &= (10 - 1)(10 + 1) \\ &= 10^2 - 1^2 \\ &= 99 \end{aligned}$$

$$\begin{aligned} 13 \times 7 &= (10 + 3)(10 - 3) \\ &= 10^2 - 3^2 \\ &= 91 \end{aligned}$$

mental workout

hard

$$\begin{aligned} 43 \times 37 &= (40 + 3)(40 - 3) \\ &= 40^2 - 3^2 \\ &= 1591 \end{aligned}$$

$$\begin{aligned} 51 \times 69 &= (60 - 9)(60 + 9) \\ &= 60^2 - 9^2 \\ &= 3519 \end{aligned}$$

$$\begin{aligned} 102 \times 98 &= (100 + 2)(100 - 2) \\ &= 100^2 - 2^2 \\ &= 9996 \end{aligned}$$

mental workout

tricky

$$\begin{aligned}15^2 &= (15^2 - 5^2) + 5^2 \\&= (15 + 5)(15 - 5) + 25 \\&= 20 \times 10 + 25 \\&= 225\end{aligned}$$

$$\begin{aligned}25^2 &= (25^2 - 5^2) + 5^2 \\&= (25 + 5)(25 - 5) + 25 \\&= 30 \times 20 + 25 \\&= 625\end{aligned}$$

mental workout

advanced
factorization

quadratic binomials

$$\begin{aligned}2x^2 - 6 &= 2(x^2 - 3) \\&= 2((x)^2 - (\sqrt{3})^2) \\&= 2(x + \sqrt{3})(x - \sqrt{3})\end{aligned}$$

$$\begin{aligned}24 - 27x^2 &= 3(8 - 9x^2) \\&= 3((\sqrt{8})^2 - (3x)^2) \\&= 3((2\sqrt{2})^2 - (3x)^2) \\&= 3(2\sqrt{2} + 3x)(2\sqrt{2} - 3x)\end{aligned}$$

harder examples

concept extension

$$\begin{aligned}(x - 3)^2 - 36 \\&= (x - 3)^2 - (6)^2 \\&= ((x - 3) + 6)((x - 3) - 6) \\&= (x + 3)(x - 9)\end{aligned}$$

$$\begin{aligned}(x - 2)^2 - 25 \\&= (x - 2)^2 - (5)^2 \\&= ((x - 2) + 5)((x - 2) - 5) \\&= (x + 3)(x - 7)\end{aligned}$$

we can still write as squares
and factor with the difference
of squares

harder examples

advanced
factorization

completing
the square

factorable expressions

We can factor any factorable quadratic expression

$$ax^2 + bx + c$$

by the difference of squares.

a, b, c are real numbers
with $a \neq 0$ and the
discriminant $b^2 - 4ac \geq 0$.

advanced factorization

completing the square

$$x^2 + 6x - 7$$
$$= (x^2 + 6x \quad) \quad - 7$$

a simple case, but we will complete the square to show how we can factor any factorable quadratic expression with this approach

put () round the x related terms and leave space before the next step of completing the square

advanced factoring

...

$$x^2 + 6x - 7$$

$$= (x^2 + 6x + 9) - 9 - 7$$

$$= (x + 3)^2 - 16$$

halve the coefficient of x
from 6 to 3

square the 3 to 9 and add
and subtract

complete the square as
 $(x + 3)^2$

advanced factoring

...

$$\begin{aligned} & x^2 + 6x - 7 \\ &= (x^2 + 6x + 9) - 9 - 7 \\ &= (x + 3)^2 - 16 \\ &= (x + 3)^2 - (4)^2 \\ &= ((x + 3) + 4)((x + 3) - 4) \\ &= (x + 7)(x - 1) \end{aligned}$$

write as squares and factor
with difference of squares

advanced factoring

completing the square

$$x^2 - 2x - 1$$
$$= (x^2 - 2x \quad) \quad - 1$$

a harder case, but we will
complete the square to show
how we can factor any
factorable quadratic
expression with this approach

put () round the x related
terms and leave space before
the next step of completing
the square

advanced factoring

...

$$x^2 - 2x - 1$$

$$= (x^2 - 2x + 1) - 1 - 1$$

$$= (x - 1)^2 - 2$$

halve the coefficient of x
from (-2) to (-1)

square the (-1) to 1 and
add and subtract

complete the square as
 $(x - 1)^2$

advanced factoring

...

$$\begin{aligned} & x^2 - 2x - 1 \\ &= (x^2 - 2x + 1) - 1 - 1 \\ &= (x - 1)^2 - 2 \\ &= (x - 1)^2 - (\sqrt{2})^2 \\ &= ((x - 1) + \sqrt{2})((x - 1) - \sqrt{2}) \\ &= (x - 1 + \sqrt{2})(x - 1 - \sqrt{2}) \\ &= (x - (1 - \sqrt{2}))(x - (1 + \sqrt{2})) \end{aligned}$$

write as squares and factor
with difference of squares

advanced factoring