

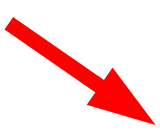
quadratic equations

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coaching
calculator

formula, factor or complete the square

Choose the method and
enter the a , b and c of
 $ax^2 + bx + c = 0$.

Choose  to enter in
descending powers of x .

e.g. enter 3, 4, and 1 for
 $3x^2 + 4x + 1 = 0$

Choose  to enter in
ascending powers of x .

e.g. enter 1, -5 , and -6
for $1 - 5x - 6x^2 = 0$

coaching calculator

terminology

terminology

quadratic equation

Standard form :

$$ax^2 + bx + c = 0 \quad a \neq 0$$

Formula :

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant :

$$D = b^2 - 4ac$$

terminology

terms and coefficients

Standard form :

$$ax^2 + bx + c = 0 \quad a \neq 0$$

The quadratic trinomial $2x^2 - 7x + 5$ has three terms $2x^2$, $-7x$ and 5 .

2 , -7 and 5 , the a , b and c , are the coefficients of the x^2 , x and constant terms (constant term is also 5).

The x term is $-7x$ with a coefficient of (-7) .

Take account of the $+$ or the $-$ operator when finding coefficients.

terminology

discriminant

Standard form :

$$ax^2 + bx + c = 0 \quad a \neq 0$$

Discriminant :

$$D = b^2 - 4ac$$

The equation has real solutions when $D \geq 0$.

We can solve all solvable quadratic equations by the quadratic formula or by completing the square.

We can solve by factoring when D is a square number (rational solutions).

terminology

common
problems

coefficients

$x^2 - x - 5 = 0$ has $a = 1$,
 $b = -1$ and $c = -5$.

We could write the equation

$$(+1)x^2 + (-1)x + (-5) = 0$$

to show coefficients clearly.

$-x^2 + x - 1 = 0$ has $a = -1$,
 $b = 1$ and $c = -1$.

We could write the equation

$$(-1)x^2 + (+1)x + (-1) = 0$$

to show coefficients clearly.

common problems

substitution

$$x^2 - x - 5 = 0$$

$$a = 1, b = -1, c = -5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-5)}}{2(1)}$$

$$-b = -(-1) = 1$$

$$b^2 = (-1)^2 = 1$$

Entering -1^2 instead of $(-1)^2$ into some calculators gives -1 not 1 .

common problems

the algebra

preparation

We start with the quadratic in decreasing powers of x .

$$ax^2 + bx + c = 0$$

Divide both sides by the coefficient of x^2 .

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Move the constant term to the right hand side.

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

We are now ready to complete the square.

the algebra

completing the square

Halve the coefficient of x

from $\frac{b}{a}$ to $\frac{b}{2a}$.

Square to $\frac{b^2}{4a^2}$ and add to both sides.

$$x^2 + \frac{b}{a}x = \frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$$

Complete the square.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2}$$

We can now square root both sides.

the algebra

square root

We square root both sides and have two potential solutions.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

the algebra

discriminant

nature of roots

The number and nature of the roots or solutions of the quadratic equation

$$ax^2 + bx + c = 0$$

depends on whether the discriminant, D , is positive, negative or zero.

$$D = b^2 - 4ac$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

discriminant

nature of roots

Discriminant $D = b^2 - 4ac$.

$$D > 0$$

two distinct real roots

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$D = 0$$

one repeated real root

$$x = -\frac{b}{2a}$$

$$D < 0$$

no real roots

discriminant