

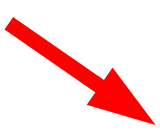
quadratic equations

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coaching
calculator

formula, factor or complete the square

Choose the method and
enter the a , b and c of
 $ax^2 + bx + c = 0$.

Choose  to enter in
descending powers of x .

e.g. enter 3, 4, and 1 for
 $3x^2 + 4x + 1 = 0$

Choose  to enter in
ascending powers of x .

e.g. enter 1, -5 , and -6
for $1 - 5x - 6x^2 = 0$

coaching calculator

terminology

terminology

quadratic polynomial

Monomial

$$x^2$$

$$6x^2$$

Binomial

$$3x^2 - 12$$

$$5x^2 + 2x$$

Trinomial

$$x^2 + 4x - 5$$

$$5x^2 - x - 6$$

terminology

terms and coefficients

Standard form :

$$ax^2 + bx + c = 0 \quad a \neq 0$$

The quadratic trinomial $2x^2 - 7x + 5$ has three terms $2x^2$, $-7x$ and 5 .

2 , -7 and 5 , the a , b and c , are the coefficients of the x^2 , x and constant terms (constant term is also 5).

The x term is $-7x$ with a coefficient of (-7) .

Take account of the $+$ or the $-$ operator when finding coefficients.

terminology

discriminant

Standard form :

$$ax^2 + bx + c = 0 \quad a \neq 0$$

Discriminant :

$$D = b^2 - 4ac$$

The equation has real solutions when $D \geq 0$.

We can solve all solvable quadratic equations by the quadratic formula or by completing the square.

We can solve by factoring when D is a square number (rational solutions).

terminology

exchanging sides

Exchange sides to take terms in descending powers of x to the left hand side.

$$6 = x^2 + 5x$$

$$x^2 + 5x = 6$$

$$x^2 + 5x - 6 = 0 \dots$$

Take to the right hand side ...

$$6 = x^2 + 5x$$

$$0 = x^2 + 5x - 6 \dots$$

... and exchange if desired.

$$6 = x^2 + 5x$$

$$0 = x^2 + 5x - 6$$

$$x^2 + 5x - 6 = 0 \dots$$

terminology

step-by-step

$$ax^2 + bx + c = 0$$

$$a, b, c \neq 0$$

descending powers

Make sure that the quadratic is written in descending powers of the variable.

Here the equation is fine.

$$4x^2 + 3x - 1 = 0$$

Here we first reorder in descending powers of x .

$$\begin{aligned} 1 - 3x - 4x^2 &= 0 \\ -4x^2 - 3x + 1 &= 0 \end{aligned}$$

We don't have to reorder, but it's a standard approach.

step-by-step

common numerical factor

Factor out the GCD or the greatest common divisor of the coefficients and divide both sides by it.

$$8x^2 + 6x - 2 = 0$$

$$2(4x^2 + 3x - 1) = 0$$

$$4x^2 + 3x - 1 = 0$$

In the next case the GCD is 1, but we factor out (-1) to end up with $4x^2$ rather than $-4x^2$.

$$1 - 3x - 4x^2 = 0$$

$$-4x^2 - 3x + 1 = 0$$

$$-(4x^2 + 3x - 1) = 0$$

$$4x^2 + 3x - 1 = 0$$

step-by-step

zero-product rule

Complete the factorization ...

$$4x^2 + 3x - 1 = 0$$

$$(x + 1)(4x - 1) = 0$$

... and then apply the zero-product rule.

$$x + 1 = 0 \text{ or } 4x - 1 = 0$$

$$x = -1 \text{ or } x = \frac{1}{4}$$

step-by-step

zero-product rule

Complete the factorization ...

$$8x^2 + 6x - 2 = 0$$

$$2(4x^2 + 3x - 1) = 0$$

$$4x^2 + 3x - 1 = 0$$

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step-by-step

zero-product rule

Complete the factorization ...

$$1 - 3x - 4x^2 = 0$$

$$-4x^2 - 3x + 1 = 0$$

$$-(4x^2 + 3x - 1) = 0$$

$$4x^2 + 3x - 1 = 0$$

$$(x + 1)(4x - 1) = 0$$

... and then apply the zero-product rule.

$$x + 1 = 0 \text{ or } 4x - 1 = 0$$

$$x = -1 \text{ or } x = \frac{1}{4}$$

We don't have to reorder, but it's a standard approach.

step-by-step

zero-product rule

Complete the factorization ...

$$2 - 6x - 8x^2 = 0$$

$$-8x^2 - 6x + 2 = 0$$

$$-2(4x^2 + 3x - 1) = 0$$

$$4x^2 + 3x - 1 = 0$$

$$(x + 1)(4x - 1) = 0$$

... and then apply the zero-product rule.

$$x + 1 = 0 \text{ or } 4x - 1 = 0$$

$$x = -1 \text{ or } x = \frac{1}{4}$$

We don't have to reorder, but it's a standard approach.

step-by-step

step-by-step

$$ax^2 + c = 0$$

$$a, c \neq 0$$

difference of squares

We can try the difference of squares if there is an x^2 term and a constant term only.

$$9x^2 - 4 = 0$$

$$(3x + 2)(3x - 2) = 0$$

Recall :

$$\begin{aligned} 9x^2 - 4 &= (3x)^2 - 2^2 \\ &= (3x + 2)(3x - 2) \end{aligned}$$

Coefficients of x^2 and constant terms have opposite sign.

There is no solution if they have the same sign.

$$9x^2 + 4 = 0$$

step-by-step

zero-product rule

Complete the factorization ...

$$9x^2 - 4 = 0$$
$$(3x + 2)(3x - 2) = 0$$

... and then apply the zero-product rule.

$$3x + 2 = 0 \text{ or } 3x - 2 = 0$$

$$x = -\frac{2}{3} \text{ or } x = \frac{2}{3}$$

step-by-step

zero-product rule

Complete the factorization ...

$$18x^2 - 8 = 0$$

$$2(9x^2 - 4) = 0$$

$$9x^2 - 4 = 0$$

$$(3x + 2)(3x - 2) = 0$$

... and then apply the zero-product rule.

$$3x + 2 = 0 \text{ or } 3x - 2 = 0$$

$$x = -\frac{2}{3} \text{ or } x = \frac{2}{3}$$

step-by-step

zero-product rule

Complete the factorization ...

$$4 - 9x^2 = 0$$

$$-9x^2 + 4 = 0$$

$$-(9x^2 - 4) = 0$$

$$9x^2 - 4 = 0$$

$$(3x + 2)(3x - 2) = 0$$

... and then apply the zero-product rule.

$$3x + 2 = 0 \text{ or } 3x - 2 = 0$$

$$x = -\frac{2}{3} \text{ or } x = \frac{2}{3}$$

We don't have to reorder, but it's a standard approach.

step-by-step

zero-product rule

Complete the factorization ...

$$8 - 18x^2 = 0$$

$$-18x^2 + 8 = 0$$

$$(-2)(9x^2 - 4) = 0$$

$$9x^2 - 4 = 0$$

$$(3x + 2)(3x - 2) = 0$$

... and then apply the zero-product rule.

$$3x + 2 = 0 \text{ or } 3x - 2 = 0$$

$$x = -\frac{2}{3} \text{ or } x = \frac{2}{3}$$

We don't have to reorder, but it's a standard approach.

step-by-step

step-by-step

$$ax^2 + bx = 0$$

$$a, b \neq 0$$

a simple factorization

We get a simple factorization with an x^2 term and x term.

Factor out an x along with the GCD ...

$$6x^2 - 12x = 0$$

$$6x(x - 2) = 0 \dots$$

We can divide both sides by **the numeric part** if desired.

$$6x^2 - 12x = 0$$

$$6x(x - 2) = 0$$

$$x(x - 2) = 0 \dots$$

step-by-step

a simple factorization

Here we write in descending powers of x and factor out a (-1) with x and the GCD.

$$12x - 6x^2 = 0$$

$$-6x^2 + 12x = 0$$

$$-6x(x - 2) = 0 \dots$$

We can divide both sides by **the numeric part** if desired.

$$12x - 6x^2 = 0$$

$$-6x(x - 2) = 0$$

$$x(x - 2) = 0 \dots$$

step-by-step

a simple factorization

Don't be tempted to divide both sides by x here.

$$x^2 = 4x$$

$$x = 4$$

Cannot divide by 0 .

Cannot divide if $x = 0$.

Lose the solution $x = 0$.

Always factor.

$$x^2 = 4x$$

$$x^2 - 4x = 0$$

$$x(x - 4) = 0 \dots$$

step-by-step