

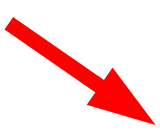
quadratic equations

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coaching
calculator

formula, factor or complete the square

Choose the method and
enter the a , b and c of
 $ax^2 + bx + c = 0$.

Choose  to enter in
descending powers of x .

e.g. enter 3, 4, and 1 for
 $3x^2 + 4x + 1 = 0$

Choose  to enter in
ascending powers of x .

e.g. enter 1, -5 , and -6
for $1 - 5x - 6x^2 = 0$

coaching calculator

terminology

terminology

quadratic polynomial

Monomial

$$x^2$$

$$6x^2$$

Binomial

$$3x^2 - 12$$

$$5x^2 + 2x$$

Trinomial

$$x^2 + 4x - 5$$

$$5x^2 - x - 6$$

terminology

terms and coefficients

Standard form :

$$ax^2 + bx + c = 0 \quad a \neq 0$$

The quadratic trinomial $2x^2 - 7x + 5$ has three terms $2x^2$, $-7x$ and 5 .

2 , -7 and 5 , the a , b and c , are the coefficients of the x^2 , x and constant terms (constant term is also 5).

The x term is $-7x$ with a coefficient of (-7) .

Take account of the $+$ or the $-$ operator when finding coefficients.

terminology

discriminant

Standard form :

$$ax^2 + bx + c = 0 \quad a \neq 0$$

Discriminant :

$$D = b^2 - 4ac$$

The equation has real solutions when $D \geq 0$.

We can solve all solvable quadratic equations by the quadratic formula or by completing the square.

We can solve by factoring when D is a square number (rational solutions).

terminology

step-by-step

step-by-step

descending powers

Make sure that the quadratic is written in descending powers of the variable.

Here the equation is fine.

$$x^2 + 3x - 4 = 0$$

Here we first reorder in descending powers of x .

$$\begin{aligned} 4 - 3x - x^2 &= 0 \\ -x^2 - 3x + 4 &= 0 \end{aligned}$$

Always check the quadratic is in descending powers of the variable.

step-by-step

division by coefficient of x^2

Divide both sides by the coefficient of x^2 .

No point dividing here. The coefficient of x^2 is 1.

$$x^2 + 3x - 4 = 0$$

Here we divide by 2.

$$2x^2 + 6x - 8 = 0$$

$$x^2 + 3x - 4 = 0$$

And here by (-2) .

$$8 - 6x - 2x^2 = 0$$

$$-2x^2 - 6x + 8 = 0$$

$$x^2 + 3x - 4 = 0$$

step-by-step

constant term to RHS

Take the constant term to the right hand side.

Here we take it immediately.

$$x^2 + 3x - 4 = 0$$

$$x^2 + 3x = 4$$

Here we write the quadratic in descending powers of x , divide both sides by (-1) , the coefficient of x^2 , and take the constant term to the RHS.

$$4 - 3x - x^2 = 0$$

$$-x^2 - 3x + 4 = 0$$

$$x^2 + 3x - 4 = 0$$

$$x^2 + 3x = 4$$

step-by-step

complete the square

Complete the square on the left hand side.

Halve the coefficient of x from 3 to $\frac{3}{2}$.

Square to $\frac{9}{4}$ and add to both sides.

Complete the square.

$$x^2 + 3x - 4 = 0$$

$$x^2 + 3x = 4$$

$$x^2 + 3x + \frac{9}{4} = 4 + \frac{9}{4}$$

$$\left(x + \frac{3}{2}\right)^2 = \frac{25}{4}$$

step-by-step

square root both sides

Square root both sides and solve.

$$x^2 + 3x - 4 = 0$$

$$x^2 + 3x = 4$$

$$x^2 + 3x + \frac{9}{4} = 4 + \frac{9}{4}$$

$$\left(x + \frac{3}{2}\right)^2 = \frac{25}{4}$$

$$x + \frac{3}{2} = \pm \sqrt{\frac{25}{4}}$$

$$x + \frac{3}{2} = \pm \frac{5}{2}$$

$$x = -\frac{3}{2} \pm \frac{5}{2}$$

$$x = -4 \text{ or } x = 1$$

step-by-step

the algebra

preparation

We start with the quadratic in decreasing powers of x .

$$ax^2 + bx + c = 0$$

Divide both sides by the coefficient of x^2 .

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Move the constant term to the right hand side.

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

We are now ready to complete the square.

the algebra

completing the square

Halve the coefficient of x

from $\frac{b}{a}$ to $\frac{b}{2a}$.

Square to $\frac{b^2}{4a^2}$ and add to both sides.

$$x^2 + \frac{b}{a}x = \frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$$

Complete the square.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2}$$

We can now square root both sides.

the algebra

square root

We square root both sides and have two potential solutions.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

the algebra

complex
solutions

negative discriminant

$$x^2 - 6x + 10 = 0$$

$$x^2 - 6x = -10$$

$$x^2 - 6x + 9 = -10 + 9$$

$$(x - 3)^2 = -1$$

$$x - 3 = \pm \sqrt{-1}$$

Discriminant :

$$D = b^2 - 4ac$$

$$= (-6)^2 - 4(1)(10)$$

$$= -4$$

$D < 0$ implies no real solution.

Completing the square in the normal way however ...

complex solutions

i

$$x^2 - 6x + 10 = 0$$

$$x^2 - 6x = -10$$

$$x^2 - 6x + 9 = -10 + 9$$

$$(x - 3)^2 = -1$$

$$x - 3 = \pm \sqrt{-1}$$

$$x - 3 = \pm i$$

$$x = 3 \pm i$$

We can define the principal square root of (-1) as i .

$$\sqrt{-1} = i$$

In this example there are two distinct complex solutions.

complex solutions