

rationalization

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coaching
calculator

data entry

$$\frac{a}{b\sqrt{c}} \quad \frac{a}{b\sqrt{c} + d} \quad \frac{a}{b + c\sqrt{d}}$$
$$\frac{a}{b\sqrt{c} + d\sqrt{e}}$$

Select template and enter a
 b , c , etc. on the number pad.

Select $\frac{a}{b\sqrt{c}}$ and enter 2, 3,
and 5 to rationalize $\frac{2}{3\sqrt{5}}$
for example.

Tap puzzled face for steps.

coaching calculator

terminology and notes

rationalization

Rationalization is the process of eliminating radicals from the denominators of expressions.

$$\frac{17}{\sqrt{15}} = \frac{17\sqrt{15}}{15}$$

$$\frac{1}{2 - \sqrt{3}} = 2 + \sqrt{3}$$

$$\frac{1}{3\sqrt{2} - 2\sqrt{3}} = \frac{3\sqrt{2} + 2\sqrt{3}}{6}$$

terminology

square

$$\frac{a}{b\sqrt{c}} = \frac{a}{b\sqrt{c}} \cdot \frac{\sqrt{c}}{\sqrt{c}} = \frac{a\sqrt{c}}{bc}$$

Multiplication by $\frac{\sqrt{c}}{\sqrt{c}}$ is
multiplication by one.

On the denominator we have
 $b\sqrt{c} \cdot \sqrt{c} = b(\sqrt{c})^2 = bc$ and
the radical has gone.

notes

difference of squares

$$\frac{1}{a + b\sqrt{c}} = \frac{1}{a + b\sqrt{c}} \cdot \frac{a - b\sqrt{c}}{a - b\sqrt{c}} \\ = \frac{a - b\sqrt{c}}{a^2 - b^2c}$$

Multiplication by $\frac{a - b\sqrt{c}}{a - b\sqrt{c}}$
is multiplication by one.

On the denominator we have
the difference of squares and
the radical has gone.

$$(a + b\sqrt{c})(a - b\sqrt{c}) \\ = a^2 - (b\sqrt{c})^2 = a^2 - b^2c$$

notes

difference of squares

$$\frac{1}{a\sqrt{b} + c\sqrt{d}}$$

$$= \frac{1}{a\sqrt{b} + c\sqrt{d}} \cdot \frac{a\sqrt{b} - c\sqrt{d}}{a\sqrt{b} - c\sqrt{d}}$$

$$= \frac{a\sqrt{b} - c\sqrt{d}}{a^2b - c^2d}$$

Multiplication by $\frac{a\sqrt{b} - c\sqrt{d}}{a\sqrt{b} - c\sqrt{d}}$

is multiplication by one.

On the denominator we have the difference of squares and the radical has gone.

$$(a\sqrt{b} + c\sqrt{d})(a\sqrt{b} - c\sqrt{d})$$

$$= (a\sqrt{b})^2 - (c\sqrt{d})^2$$

$$= a^2b - c^2d$$

notes

radical simplification

We occasionally have radicals that simplify.

$$\frac{1}{2\sqrt{2} - \sqrt{8}} = \frac{1}{2\sqrt{2} - 2\sqrt{2}}$$

Did you spot the division by zero here ?

Easy to see when we simplify radicals before rationalization.

It's advisable to simplify any radicals before rationalization so we can spot potential problems from the start.

notes

examples

examples

square

$$\begin{aligned}\frac{100}{\sqrt{11}} &= \frac{100}{\sqrt{11}} \cdot \frac{\sqrt{11}}{\sqrt{11}} \\ &= \frac{100\sqrt{11}}{(\sqrt{11})^2} \\ &= \frac{100\sqrt{11}}{11}\end{aligned}$$

$$\begin{aligned}\frac{7}{2\sqrt{5}} &= \frac{7}{2\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} \\ &= \frac{7\sqrt{5}}{2 \cdot (\sqrt{5})^2} \\ &= \frac{7\sqrt{5}}{2 \cdot 5} \\ &= \frac{7\sqrt{5}}{10}\end{aligned}$$

examples

square

$$\begin{aligned}\frac{5}{3\sqrt{8}} &= \frac{5}{3(2\sqrt{2})} \\ &= \frac{5}{6\sqrt{2}} \\ &= \frac{5}{6\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{5\sqrt{2}}{6 \cdot (\sqrt{2})^2} \\ &= \frac{5\sqrt{2}}{6 \cdot 2} \\ &= \frac{5\sqrt{2}}{12}\end{aligned}$$

examples

difference of squares

$$\begin{aligned}\frac{1}{5 + \sqrt{18}} &= \frac{1}{5 + 3\sqrt{2}} \\&= \frac{1}{(5 + 3\sqrt{2})} \cdot \frac{(5 - 3\sqrt{2})}{(5 - 3\sqrt{2})} \\&= \frac{5 - 3\sqrt{2}}{5^2 - (3\sqrt{2})^2} \\&= \frac{5 - 3\sqrt{2}}{5^2 - 3^2 \cdot (\sqrt{2})^2} \\&= \frac{5 - 3\sqrt{2}}{25 - 9 \cdot 2} \\&= \frac{5 - 3\sqrt{2}}{7}\end{aligned}$$

examples

difference of squares

$$\frac{5}{7 - 2\sqrt{3}}$$

$$= \frac{5}{(7 - 2\sqrt{3})} \cdot \frac{(7 + 2\sqrt{3})}{(7 + 2\sqrt{3})}$$

$$= \frac{5(7 + 2\sqrt{3})}{7^2 - (2\sqrt{3})^2}$$

$$= \frac{5(7 + 2\sqrt{3})}{7^2 - 2^2 \cdot (\sqrt{3})^2}$$

$$= \frac{5(7 + 2\sqrt{3})}{49 - 4 \cdot 3}$$

$$= \frac{35 + 10\sqrt{3}}{37}$$

examples

difference of squares

$$\frac{1}{3\sqrt{2} + 2\sqrt{3}}$$

$$= \frac{1}{(3\sqrt{2} + 2\sqrt{3})} \cdot \frac{(3\sqrt{2} - 2\sqrt{3})}{(3\sqrt{2} - 2\sqrt{3})}$$

$$= \frac{3\sqrt{2} - 2\sqrt{3}}{(3\sqrt{2})^2 - (2\sqrt{3})^2}$$

$$= \frac{3\sqrt{2} - 2\sqrt{3}}{3^2 \cdot (\sqrt{2})^2 - 2^2 \cdot (\sqrt{3})^2}$$

$$= \frac{3\sqrt{2} - 2\sqrt{3}}{9 \cdot 2 - 4 \cdot 3}$$

$$= \frac{3\sqrt{2} - 2\sqrt{3}}{6}$$

examples